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Intersonic crack growth under time-dependent loading

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Abstract

The problem investigated in this paper is a mode II crack extending at a uniform intersonic speed in an otherwise unbounded elastic solid subjected to time dependent crack-face tractions. The fundamental solution for this problem is obtained analytically, which is then used to construct the general solution for an intersonic crack subjected to arbitrary time-dependent loading. For time-independent loading, this solution reduces to Huang and Gao's [J. Appl. Mech 68 (2001) 169] fundamental solution. We have also studied two crack-face loadings that are of interest for engineering applications.
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1. Introduction

Indirect evidence of shear crack propagation in excess of shear wave speed was reported from observations of shallow crustal earthquakes (Archuleta, 1982; Beroza and Spudich, 1988; Wald and Heaton, 1994; Ellsworth and Celebi, 1999). However, it was the experiment of Rosakis et al. (1999) that stimulated the resurgence of the recent studies on intersonic crack growth. The experiment provided a direct laboratory observation of shear-dominated intersonic crack propagation along a weak plane in an otherwise homogeneous polyester resin. The test indicated a crack propagation velocity faster than shear wave speed c_s and even close to longitudinal wave speed c_l of the material.

The elastic response is likely to dominate in the case of intersonic crack growth. The elasticity studies (Burridge, 1973; Andrews, 1976; Brock, 1977; Burridge et al., 1979; Freund, 1979; Simonov, 1983; Broberg, 1989, 1994, 1999; Freund, 1990; Gao et al., 1999; Huang et al., 1999; Yu and Suo, 2000) in the past 30 years have fully explored the possibility of intersonic crack growth. The cracking mode dictates the possibility of intersonic crack growth. No intersonic cracking regime exists for the mode III case. With only one (shear) wave speed in anti-plane shear, the crack propagation under anti-plane shear is either subsonic or supersonic. For a mode I crack, the physically admissible stress singularity and the energy release rate vanish for

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all crack velocities above the Rayleigh wave speed, rendering a forbidden zone that covers the complete intersonic and supersonic regimes. That leaves mode II cracking as the only candidate of intersonic crack growth. For a mode II crack in an elastic medium, the stress singularity of intersonic cracks is less than 1/2 except at a special (radiation-free) crack-tip velocity of $\sqrt{2}c_s$, at which the energy release rate is finite and non-vanishing. By adopting a finite cohesive zone extended from the crack tip, the crack-tip energy release rate may remain positive for all intersonic speeds. Numerical studies on intersonic crack propagation by finite element simulation or atomistic simulation (Needleman, 1999; Needleman and Rosakis, 1999; Abraham and Gao, 2000; Gao et al., 2001; Geubelle and Kubair, 2001) confirmed the possibility of intersonic crack growth under mode II case.

In all of those researches, the crack tip is confined to propagate along a predetermined straight-line path. Without the confine, the mode I crack tends to oscillate and eventually branch at a specific propagation speed, namely, only 0.35–0.5 c_R. Rather than propagating straight ahead, an initially mode II crack would curve continuously, or if necessary, kink abruptly to ensure that it remains a locally mode-I crack. The natural tendency of growing cracks to propagate under strictly mode-I conditions in homogeneous monolithic solids explains the lack of interest of early engineering researchers in mixed mode, or mode-II, dynamic crack growth. In recent years this situation has changed drastically since there is an increasing demand for specialized lightweight, high-strength structures made out of inhomogeneous (heterogeneous) solids. Such solids include structural composites sandwich structures, bonded layered materials, and continuously graded solids. Many of these materials contain weak paths for crack debonding. If the fracture mode along those weak paths is mode II in nature, a possibility of intersonic crack growth emerges. The mode II cracks propagating along the predetermined path also exist in earthquake due to the high pressure and the fault.

Until recently, most analytical studies were focused on the possibility of intersonic crack growth. Only the simplest situations such as steady state or self-similar crack growth were explored. Seldom studied is how loading history modulates the growth of an intersonic crack. Huang and Gao (2001) gave the transient fundamental solution for an initial stationary crack propagating at a steady intersonic crack tip speed under time independent loading. The solution can be utilized to construct the general solution for uniform intersonic crack propagation subjected to an arbitrary initial equilibrium field. Antipov and Willis (2003) obtained the fundamental solution for intersonic crack propagation in linear viscous solids. Huang and Gao (2002) obtained the solution for a suddenly arrested crack that previously propagated at an intersonic speed. The recent work of Guo et al. (2003) investigated suddenly decelerating or accelerating intersonic cracks. Both works showed that the stress intensity factor does not instantaneously reach its equilibrium value when an intersonically propagating crack tip changes its speed.

In this paper, uniform intersonic crack growth in an elastic medium is considered with attention focused on time-dependent loading. The restriction on an intersonic cracking regime leaves the mode II case as the only scenario to be considered. The fundamental solution is obtained in the same spirit as in Guo et al. (2003). This fundamental solution for time-dependent loading is then used to construct the general solution for arbitrary crack-face loadings. For the limiting case of time independent loading, our fundamental solution reduces to the result obtained by Huang and Gao (2001). The solutions are also presented for several time-dependent crack-face tractions. The method can also be applied straightforwardly to subsonic crack growth, which has been extensively studied (e.g., Freund, 1972, 1990; Kostrov, 1975; Saraikin and Slepyan, 1979; Willis, 1989; Slepyan, 2002).

2. Crack growth due to time-dependent loading

Consider a semi-infinite crack in an otherwise unbounded body. The material is linear elastic and isotropic, with shear modulus μ, Poisson's ratio ν, shear wave speed c_s and longitudinal wave speed c_l. A plane strain situation is assumed. The material surrounding the crack tip is initially free of loading. At time $t = 0$, the crack tip region is stressed due to either the passage of a stress wave or the sudden application of crack-face tractions. Assume the loading is skew-symmetric so that the crack tip field is mode II. At the same instant, the crack begins to extend at a constant intersonic speed $\upsilon(c_s \leqslant \upsilon \leqslant c_l)$ within the crack plane.

The applied load can be viewed as a series of concentrated forces that begin to act on the crack surface at different time. We first study a pair of concentrated shear forces on each side of the crack surfaces. The forces are applied at an arbitrary location behind the crack tip, and their position and magnitude remain unchanged thereafter. This solution is called the fundamental solution because the solution for more general loadings can be constructed by its superposition.

The intersonic crack tip has a stress singularity $-q$ that is generally different from the conventional square-root singularity $-1/2$, where q is a function of the crack tip velocity υ and is given by

$$q = \frac{1}{\pi}\tan^{-1}\frac{4\alpha_l\hat{\alpha}_s}{\left(2-\frac{\upsilon^2}{c_s^2}\right)^2} = \frac{1}{\pi}\tan^{-1}\frac{4\alpha_l\hat{\alpha}_s}{(1-\hat{\alpha}_s^2)^2}, \tag{1}$$

and

$$\alpha_l = \left(1-\frac{\upsilon^2}{c_l^2}\right)^{1/2}, \quad \hat{\alpha}_s = \left(\frac{\upsilon^2}{c_s^2}-1\right)^{1/2}. \tag{2}$$

The focus of our study is on the stress intensity factor for intersonic crack growth, which is defined as $k_{II} = \lim_{r\to 0}\sqrt{2\pi}r^q\sigma_{xy}\big|_{\theta=0}$, where r and θ are the polar coordinates measured from the crack tip and $\theta = \pm\pi$ are the crack faces.

2.1. Fundamental solution

The x–y coordinate system is introduced such that the crack lies along the x-axis and grows toward the positive x direction at an intersonic speed υ. At time $t = 0$, the crack tip passes the point $(l, 0)$, and a pair of opposite unit concentrated shear forces $\delta(x)H(t)$ on the upper and lower crack faces begin to act at the origin.

Only the upper half plane is considered due to the skew-symmetry. The application of the concentrated force produces an elastic wave whose front moves at the longitudinal wave speed c_l. Before the arrival of the wave front, namely $t < l/(c_l - \upsilon)$, the crack tip is stress free. The elastic field assembles that of a concentrated force acts on the surface of an elastic half plane, which is a special case of the Lamb problem (Lamb, 1904). Using the method of integral transform, one arrives at the following surface displacement in x direction (Achenbach, 1973)

$$u_x^L(x,t) = -\frac{1}{\pi\mu c_s^2}\int_{1/c_l}^{t/|x|}\mathrm{Im}\left[\frac{(c_s^{-2}-r^2)^{1/2}}{4r^2(c_l^{-2}-r^2)^{1/2}(c_s^{-2}-r^2)^{1/2}+(c_s^{-2}-2r^2)^2}\right]\mathrm{d}rH(c_lt-|x|), \tag{3}$$

where Im stands for the imaginary part of a complex argument, and H is the Heaviside step function. The above expression can be equivalently written as

$$u_x^L(x,t) = \frac{1}{\pi\mu c_s^2}\int_{1/c_l}^{t/x}\frac{4r^2(c_s^{-2}-r^2)(r^2-c_l^{-2})^{1/2}-(c_s^{-2}-2r^2)^2(r^2-c_s^{-2})^{1/2}H(r-c_s^{-1})}{16r^4(r^2-c_l^{-2})(c_s^{-2}-r^2)+(c_s^{-2}-2r^2)^4}\mathrm{d}r \tag{4}$$

for $0 < x < c_lt$.

Apparently, u_x^L depends on time t and coordinate x only through their ratio $\bar{\upsilon} = x/t$. This implies that any given displacement level radiates out along the x-axis at a constant speed $\bar{\upsilon}$. The above displacement can then be considered as a series of dislocations with the (half) Burgers vector $\mathrm{d}u_x^{\mathrm{L}}(\bar{v})$ moving with the velocity $\bar{v}$. The derivative of u_x^{L} with respect to $\bar{v}$ is given by

$$\frac{\mathrm{d}u_x^{\mathrm{L}}(\bar{v})}{\mathrm{d}\bar{v}} = -\frac{1}{\pi\mu c_{\mathrm{s}}^2\bar{v}^2}\frac{4v^{-2}(c_{\mathrm{s}}^{-2}-\bar{v}^{-2})(\bar{v}^{-2}-c_{\mathrm{l}}^{-2})^{1/2}-(c_{\mathrm{s}}^{-2}-2\bar{v}^{-2})^2(\bar{v}^{-2}-c_{\mathrm{s}}^{-2})^{1/2}H(c_{\mathrm{s}}-\bar{v})}{16\bar{v}^{-4}(\bar{v}^{-2}-c_{\mathrm{l}}^{-2})(c_{\mathrm{s}}^{-2}-\bar{v}^{-2})+(c_{\mathrm{s}}^{-2}-2\bar{v}^{-2})^4} \quad (5)$$

for $0 < \bar{v} < c_{\mathrm{l}}$.

When the longitudinal wave catches up with the crack tip, wave will be diffracted from the crack tip. The Lamb problem is not the solution ahead of the intersonic crack tip anymore because the above surface displacement does not satisfy the vanishing displacement condition, $u_x(x > l + vt, y = 0) = 0$. Similar to the approach for a suddenly arrested crack (Huang and Gao, 2002) and for a suddenly decelerating crack (Guo et al., 2003), we use the method of superposition to negate the above surface displacement ahead of the intersonic crack tip. Let us assume the wave in the Lamb problem continues to propagate in its incipient form, producing a displacement u_x^{L} in the x direction ahead of the crack tip. To negate u_x^{L}, a series of moving edge dislocations are superposed at the propagating crack tip. A dislocation that has the (half) Burgers vector $\mathrm{d}u_x^{\mathrm{L}}(\bar{v})$ is emitted from the propagating crack tip at time $l/(\bar{v}-v)$ and moves along the positive x direction in the upper half plane with a velocity $\bar{v}$, where $\bar{v}$ is between $v + l/t$ and c_{l}. The same process, namely the Lamb problem and a dislocation of the same (half) Burgers vector $\mathrm{d}u_x^{\mathrm{L}}(\bar{v})$ emitted from the crack tip, can be applied to the lower half plane. The Lamb problems in the upper and the lower half planes are skew-symmetric with respect to the crack extension line, so are the problems involving two (half) Burgers vectors propagating with the same velocity. With the skew-symmetry in mind, one may only consider the upper half plane. Such a sequence of dislocation emission indeed negates the displacement u_x^{L} ahead of the intersonic crack tip.

Guo et al. (2003) obtained the solution for an edge dislocation with the unit (half) Burgers vector being emitted from an intersonic shear crack tip at time $t = 0$ and then moving with velocity $\bar{v}$ faster than the crack tip speed v. The stress intensity factor around the intersonic crack tip propagating with the velocity $v(< \bar{v})$ is given by

$$k_0^{\mathrm{int}}(t,\bar{v},v) = -4\mu\sqrt{\frac{2}{\pi}}\alpha_{\mathrm{l}}\hat{\alpha}_{\mathrm{s}}\frac{c_{\mathrm{s}}^3(c_{\mathrm{l}}-v)}{v^2(v^2-c_{\mathrm{R}}^2)}f(v)\left(\frac{v^2-c_{\mathrm{s}}^2}{c_{\mathrm{l}}^2-v^2}\right)^q\frac{\bar{v}^2-c_{\mathrm{R}}^2}{\sqrt{\bar{v}+c_{\mathrm{l}}}\sqrt{\bar{v}^2-c_{\mathrm{s}}^2}\sqrt{\bar{v}-v}}\frac{s_-[-1/(\bar{v}-v)]}{s_-(0)}[(c_{\mathrm{l}}-v)t]^{q-1}, \quad (6)$$

where the dependence on the time t, dislocation velocity $\bar{v}$ and crack tip speed v is shown explicitly, c_{R} is the Rayleigh wave speed, and the functions $(s_-(\zeta))/(s_-(0))$ and $f(v)$ are given by

$$\frac{s_-(\zeta)}{s_-(0)} = \exp\left\{-\frac{\zeta}{\pi}\int_{1/(c_{\mathrm{l}}+v)}^{+\infty}\left[\frac{\pi}{2}+\left(\frac{\pi}{2}-\tan^{-1}V_-(r,v)\right)\left(H\left(\frac{1}{v+c_{\mathrm{s}}}-r\right)-H\left(r-\frac{1}{v-c_{\mathrm{s}}}\right)\right)\right]\frac{\mathrm{d}r}{r(r-\zeta)}\right\}, \quad (7)$$

$$f(v) = \exp\left\{\left[\int_{1/(c_{\mathrm{l}}+v)}^{1/(c_{\mathrm{s}}+v)} - \int_{1/(v-c_{\mathrm{s}})}^{+\infty}\right]\tan^{-1}\left[\frac{4\alpha_{\mathrm{l}}\hat{\alpha}_{\mathrm{s}}V_-(r,v)-\left(2-\frac{v^2}{c_{\mathrm{s}}^2}\right)^2}{4\alpha_{\mathrm{l}}\hat{\alpha}_{\mathrm{s}}+\left(2-\frac{v^2}{c_{\mathrm{s}}^2}\right)^2V_-(r,v)}\right]\frac{\mathrm{d}r}{\pi r}\right\}, \quad (8)$$

and

$$V_-(r,\upsilon)=\frac{\left[2r^2-\frac{\upsilon^2}{c_s^2}\left(r-\frac{1}{\upsilon}\right)^2\right]^2}{4r^2\sqrt{1-\frac{\upsilon^2}{c_l^2}}\sqrt{\frac{\upsilon^2}{c_s^2}-1}\sqrt{\left(r+\frac{1}{c_l-\upsilon}\right)\left(r-\frac{1}{c_l+\upsilon}\right)\left|r-\frac{1}{\upsilon-c_s}\right|\left|r-\frac{1}{\upsilon+c_s}\right|}}. \tag{9}$$

When a dislocation is emitted from the intersonic crack tip at time $l/(\bar{\upsilon}-\upsilon)$ (instead of $t=0$) with the velocity $\bar{\upsilon}$ and (half) Burgers vector $\mathrm{d}u_x^{\mathrm{L}}(\bar{\upsilon})$, the crack tip stress intensity factor is $k_0^{\mathrm{int}}[t-l/(\bar{\upsilon}-\upsilon),\bar{\upsilon},\upsilon]\,\mathrm{d}u_x^{\mathrm{L}}(\bar{\upsilon})$.

The stress intensity factor K_{F} of the fundamental solution can be obtained by superposing the Lamb solution and the solution for all dislocations. Since the Lamb solution has no stress singularity at the propagating crack tip, the stress intensity factor K_{F} is given by

$$K_{\mathrm{F}}(t,\upsilon,l)=\int_{\upsilon+\frac{l}{t}}^{c_l}k_0^{\mathrm{int}}\left(t-\frac{l}{\bar{\upsilon}-\upsilon},\omega,\bar{\upsilon}\right)\frac{\mathrm{d}u_x^{\mathrm{L}}(\bar{\upsilon})}{\mathrm{d}\bar{\upsilon}}\mathrm{d}\bar{\upsilon}H\left(t-\frac{l}{c_l-\upsilon}\right), \tag{10}$$

where the dependence on the time t, crack tip velocity υ and location of the concentrated load l is shown explicitly. Substituting (5) and (6) into the above expression, and after some lengthy algebra, one has

$$\begin{aligned}K_{\mathrm{F}}(t,\upsilon,l)=&\frac{16}{\pi}\sqrt{\frac{2}{\pi}}c_lc_s\frac{\alpha_l\hat{\alpha}_sf(\upsilon)}{(\upsilon^2-c_{\mathrm{R}}^2)\upsilon^2}\left(\frac{\upsilon^2-c_s^2}{c_l+\upsilon}\right)^qH\left(t-\frac{l}{c_l-\upsilon}\right)\\&\times\int_{\upsilon+\frac{l}{t}}^{c_l}\frac{s_-\left(-\frac{1}{\bar{\upsilon}-\upsilon}\right)}{s_-(0)}\frac{\sqrt{c_l-\bar{\upsilon}}}{\sqrt{\bar{\upsilon}-\upsilon}}\frac{\bar{\upsilon}\sqrt{\bar{\upsilon}^2-c_s^2}(\bar{\upsilon}^2-c_{\mathrm{R}}^2)\left(t-\frac{l}{\bar{\upsilon}-\upsilon}\right)^{q-1}}{16(c_l^2-\bar{\upsilon}^2)(\bar{\upsilon}^2-c_s^2)+c_l^2c_s^2\left(2-\frac{\bar{\upsilon}^2}{c_s^2}\right)^4}\mathrm{d}\bar{\upsilon}.\end{aligned} \tag{11}$$

Extensive numerical evaluations indicate that the above stress intensity factor for the limit $l=0$ (or equivalently for $t\gg l/(c_l-\upsilon)$) reduces to Huang and Gao's (2001) fundamental solution for time independent loading.

Fig. 1 shows the normalized crack tip stress intensity factor $K_{\mathrm{F}}l^{1-q}$ versus normalized time c_st/l. The concentrated shear force has a unit magnitude (of one) such that $K_{\mathrm{F}}l^{1-q}$ is dimensionless. The Poisson's ratio is $v=1/3$, and consequently the longitudinal wave speed of $c_l=2c_s$ and a Rayleigh wave speed of $c_{\mathrm{R}}=0.93c_s$. The crack growth velocity is taken as $\upsilon=1.4c_s$ in Fig. 1. The stress intensity factor remains zero until the longitudinal wave catches up the intersonic crack tip at the normalized time $c_st/l=1.67$. The stress intensity factor then starts to increase from zero, reaches a peak value at time $c_st/l=3.5$ and gradually diminishes to zero as the crack tip moves away and leaves the point force far behind. In fact, for $c_st/l\gg 1$, our fundamental solution approaches Huang and Gao's (2001).

2.2. Arbitrary time-dependent crack-face loading

We investigate the stress intensity factor around an intersonic crack tip subjected to an arbitrary crack-face shear loading. The material remains stationary and the crack tip is at the origin $x=0$, $y=0$ for time $t\leqslant 0$. The equal and opposite general shear stress tractions $T(x,t)$ (with $T(x,t\leqslant 0)=0$) are imposed on the crack faces $(x<0,y=0)$ at time $t=0$. At the same instant $t=0$ the crack tip starts to propagate at a constant intersonic speed υ towards the positive x direction. The shear stress traction acting on the crack faces can be viewed as a series of time-independent shear concentrated forces that begin to act on the crack faces at the appropriate time. This is because

$$T(x,t)=\int_0^t\int_{-\infty}^{\upsilon t_0}\frac{\mathrm{d}T(x_0,t_0)}{\mathrm{d}t_0}\delta(x-x_0)H(t-t_0)\,\mathrm{d}x_0\,\mathrm{d}t_0. \tag{12}$$

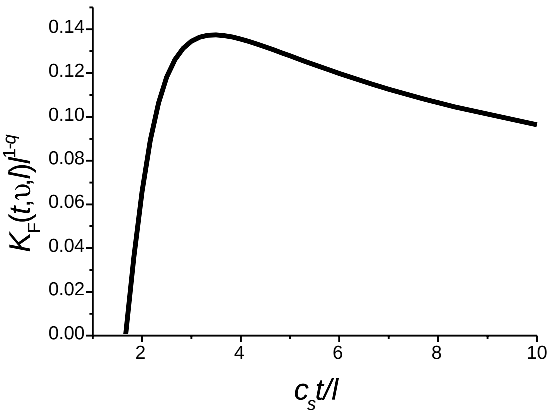

Fig. 1. The normalized stress intensity factor $K_F l^{1-q}$, versus the normalized time $c_s t/l$, in the fundamental solution for an intersonic crack subjected to time-dependent loading. An initially stationary crack starts to propagate intersonically at time $t = 0$, a pair of concentrated shear force is applied at a distance l behind the crack tip on the crack faces at the same instant. Poisson's ratio $\nu = 0.3$, c_s is the shear wave speed, and the crack tip velocity $\upsilon = 1.4c_s$.

In this equation, $(\mathrm{d}T(x_0,t_0)/\mathrm{d}t_0)\delta(x-x_0)H(t-t_0)$ is a concentrated force that acts at the point $(x_0, y=0)$ after $t = t_0$. The fundamental solution for such a concentrated force has been obtained in Section 2.1. Integrating the fundamental solution for different (x_0, t_0) gives the solution for crack growth due to time-dependent loading $T(x,t)$. Specifically, the stress intensity factor is given by

$$K(t) = \int_0^t \int_{-\infty}^{\upsilon t_0} \frac{\mathrm{d}T(x_0,t_0)}{\mathrm{d}t_0} K_F(t-t_0, \upsilon, \upsilon t_0 - x_0)\,\mathrm{d}x_0\,\mathrm{d}t_0, \tag{13}$$

where K_F is given in (10). For a crack-face loading T that starts from a finite value at time $t = 0$ (e.g., suddenly applied crack-face shear), T has a finite jump from $T = 0$ at time $t = 0^-$ to $T(x, 0^+)$ at time $t = 0^+$. Accordingly (13) becomes

$$K(t) = \int_{-\infty}^0 T(x_0, 0^+) K_F(t, \upsilon, -x_0)\,\mathrm{d}x_0 + \int_0^t \int_{-\infty}^{\upsilon t_0} \frac{\mathrm{d}T(x_0,t_0)}{\mathrm{d}t_0} K_F(t-t_0, \upsilon, \upsilon t_0 - x_0)\,\mathrm{d}x_0\,\mathrm{d}t_0. \tag{14}$$

3. Two representative crack-face loadings

We present two examples of the crack-face loading to illustrate the analytical solution given in (13) and (14).

3.1. An initial equilibrium field

Consider a semi-infinite mode II crack in an otherwise unbounded solid. The solid is subjected to an initial equilibrium field before time $t = 0$ and the crack tip is at the origin. At time $t = 0$ the crack begins to extend towards the positive x direction at a constant speed υ in the crack plane. Freund (1990) pointed out the process of crack propagation is essentially the negation of the equilibrium traction distribution during dynamic crack growth, and obtained the fundamental solution for subsonic crack propagation under time-independent loading. Huang and Gao (2001) obtained the fundamental solution for intersonic crack propagation.

The stress ahead of the crack tip in the equilibrium field is denoted by $\tau^*(x)$. This problem can be viewed as the superposition of two sub-problems: (i) a static crack with the equilibrium stress distribution $\tau^*(x)$ ahead of the crack tip; and (ii) a crack propagating with an intersonic speed υ and subjected to the negating crack-face traction $-\tau^*(x)$. Therefore the $T(x_0, t_0)$ term in Eq. (13) is

$$T(x_0, t_0) = \tau^*(x_0)H(\upsilon t_0 - x_0)H(x_0). \tag{15}$$

Its substitution into Eq. (13) gives the stress intensity factor of intersonic shear crack subjected to initial equilibrium field as

$$K(t) = \int_0^t \tau^*(\upsilon t_0)K_{\mathrm{F}}(t - t_0, \upsilon, 0)\,\upsilon\,\mathrm{d}t_0. \tag{16}$$

An example of the initial equilibrium field is the classical, static K field, i.e., $\tau^*(x_0) = K_0/\sqrt{2\pi x_0}$, where K_0 is the static stress intensity factor before crack propagation. The substitution of the above $\tau^*(x_0)$ into Eq. (16) gives the dynamic stress intensity factor around an intersonic crack tip as $K = K_0 k(\upsilon)(c_s t)^{q-1/2}$, where $k(\upsilon)$ is a non-dimensional function of the crack tip velocity υ and is on the order of 1. It is clearly seen that, as time increases, the dynamic stress intensity factor decrease monotonically (unless $q = 1/2$ for $\upsilon = \sqrt{2}c_s$) such that the intersonic crack growth is not sustainable under the initial classical, static K field.

3.2. Time-dependent concentrated load

Consider a semi-infinite crack along the negative x-axis, with the crack tip located once again at the origin for time $t \leqslant 0$. A pair of time-dependent concentrated forces is applied at the origin at time $t = 0$, and its magnitude changes with time from zero. The term $T(x_0, t_0)$ in Eq. (13) takes the form

$$T(x_0, t_0) = \delta(x_0)H(t_0)g(t_0), \tag{17}$$

where $g(0) = 0$.

The stress intensity factor is obtained from (13) as

$$K = \frac{16}{\pi}\sqrt{\frac{2}{\pi}}\frac{\alpha_1\hat{\alpha}_s c_s c_1}{\upsilon^2(\upsilon^2 - c_R^2)}f(\upsilon)\left(\frac{\upsilon^2 - c_s^2}{c_1 + \upsilon}\right)^q \int_\upsilon^{c_1} \frac{S_-\left(-\frac{1}{\bar{\upsilon}-\upsilon}\right)}{S_-(0)} \times \frac{(\bar{\upsilon}^2 - c_R^2)\bar{\upsilon}\sqrt{\bar{\upsilon}^2 - c_s^2}\sqrt{c_1 - \bar{\upsilon}}}{\sqrt{\bar{\upsilon} - \upsilon}\left[16(c_1^2 - \bar{\upsilon}^2)(\bar{\upsilon}^2 - c_s^2) + c_1^2 c_s^2\left(2 - \frac{\bar{\upsilon}^2}{c_s^2}\right)^4\right]} \int_0^{t\frac{\bar{\upsilon}-\upsilon}{\bar{\upsilon}}} \left(t - \frac{\bar{\upsilon}t_0}{\bar{\upsilon} - \upsilon}\right)^{q-1} \dot{g}(t_0)\,\mathrm{d}t_0\,\mathrm{d}\bar{\upsilon}. \tag{18}$$

Let us consider the special case where function $g(t_0)$ changes in proportion to the power of t_0, namely $g(t_0) = At_0^a$ ($a > 0$), with A and a being parameters characterizing the strength and time history of the point force loading. Accordingly, the time integration in Eq. (18) can be evaluated explicitly as

$$K = \frac{16}{\pi}\sqrt{\frac{2}{\pi}}\frac{\alpha_1\hat{\alpha}_s c_s c_1}{\upsilon^2(\upsilon^2 - c_R^2)}f(\upsilon)\left(\frac{\upsilon^2 - c_s^2}{c_1 + \upsilon}\right)^q Aa\frac{\Gamma(q)\Gamma(a)}{\Gamma(q + a)}t^{q+a-1} \times \int_\upsilon^{c_1} \frac{S_-\left(-\frac{1}{\bar{\upsilon}-\upsilon}\right)}{S_-(0)} \frac{(\bar{\upsilon}^2 - c_R^2)\sqrt{\bar{\upsilon}^2 - c_s^2}\sqrt{c_1 - \bar{\upsilon}}\bar{\upsilon}^{1-a}(\bar{\upsilon} - \upsilon)^{a-1/2}}{16(c_1^2 - \bar{\upsilon}^2)(\bar{\upsilon}^2 - c_s^2) + c_1^2 c_s^2\left(2 - \frac{\bar{\upsilon}^2}{c_s^2}\right)^4}\mathrm{d}\bar{\upsilon}. \tag{19}$$

The physical significance of this solution is an appreciation for the effect on the stress intensity factor for an intersonically propagating crack by the time history of the applied point force. When the power index a is greater than $1 - q$, the strength of the crack tip stress intensity factor K is monotonically increasing.

When the power index a is less than $1-q$, the strength of the crack tip stress intensity factor K is monotonically decreasing. The most interesting case is for maintaining the power index a as $1-q$. The stress intensity factor K becomes constant, ideal for steady intersonic crack growth

$$K=\frac{16}{\pi}\sqrt{\frac{2}{\pi}}\frac{\alpha_1\hat{\alpha}_s c_s c_1}{\upsilon^2(\upsilon^2-c_R^2)}f(\upsilon)\left(\frac{\upsilon^2-c_s^2}{c_1+\upsilon}\right)^q Aa\frac{\pi}{\sin(q\pi)}$$
$$\times\int_{\upsilon}^{c_1}\frac{s_-\left(-\frac{1}{\bar{\upsilon}-\upsilon}\right)}{s_-(0)}\frac{(\bar{\upsilon}^2-c_R^2)\sqrt{\bar{\upsilon}^2-c_s^2}\sqrt{c_1-\bar{\upsilon}}\bar{\upsilon}^q(\bar{\upsilon}-\upsilon)^{1/2-q}}{16(c_1^2-\bar{\upsilon}^2)(\bar{\upsilon}^2-c_s^2)+c_1^2c_s^2\left(2-\frac{\bar{\upsilon}^2}{c_s^2}\right)^4}\mathrm{d}\bar{\upsilon}. \tag{20}$$

That would provide a loading pattern at a fixed loading point to maintain a constant stress intensity factor for the complete history of the intersonic crack propagation. Since the intersonic crack singularity q depends on the speed and is peaked at $\sqrt{2}c_s$, the power index q to maintaining constant stress intensity factor also depends on the propagation speed. The farther the cracking speed is from $\sqrt{2}c_s$, the smaller the value of q, and the larger the required value of power index a to maintain constant stress intensity factor.

4. Concluding remarks

We have achieved the following goals in the present study.

I. We have obtained the fundamental solution for intersonic shear crack propagation subjected to arbitrary time-dependent crack-face loadings. This is achieved by the superposition of the Lamb problem for a concentrated force on the boundary of a half plane and the moving dislocation solution.
II. For the arbitrary, time-dependent loading on faces of an intersonic crack, we have found the analytic expression of the stress intensity factor for the intersonic shear crack. Several examples of the crack-face loadings are presented to illustrate our approach and the results.

It should be pointed out that the proposed approach to superpose the Lamb problem with the moving dislocation solution can also be applied straightforwardly to subsonic crack propagation.

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